Mathematical Applications in Engineering and Science

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Graduate Student Reading Seminar

Week 1: Introduction to Topics

Proposed Topics for Consideration

Mechanical and Aerospace Engineering

Quantum Computing

Artificial Intelligence and Machine Learning

Mathematical Biology

Why Mechanical and Aerospace Engineering?

- Critical for advancing technology in transportation, energy, and exploration.
- Applications span:
  - Aircraft and spacecraft design.
  - Renewable energy systems like wind turbines.
  - Autonomous vehicles and drones.

Governing Equations:

- **Navier-Stokes Equations:** Describes fluid motion.

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u}\right) = -\nabla p + \mu\nabla^2\mathbf{u} + \mathbf{f}$$

- **Heat Transfer Equation:** Describes thermal energy distribution.

$$\frac{\partial T}{\partial t} - \alpha\nabla^2 T = Q$$

Mathematical Foundations in MAE

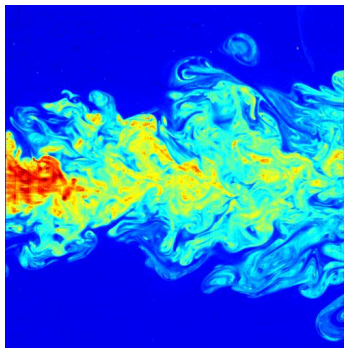

Figure: Flow visualization of a turbulent jet, made by laser-induced fluorescence. The jet exhibits a wide range of length scales, an important characteristic of turbulent flows.

- **Numerical Methods:**
  - Finite Element Analysis (FEA) for structural mechanics.
  - Computational Fluid Dynamics (CFD) for flow simulation.
- **Millennium Prize Problem:**
  - Prove the existence and smoothness of solutions in three dimensions.
  - One of the seven Millennium Prize Problems, with a $1,000,000$ dollar reward for a solution!

Quantum Computing: Revolutionizing Computation

- **What is Quantum Computing?**
  - Harnesses principles of quantum mechanics (superposition, entanglement).
  - Operates using quantum bits (qubits), capable of existing in multiple states simultaneously.
- **Why is it Important?**
  - Solutions for problems like factoring large numbers (Shor's algorithm) and unstructured search (Grover's algorithm).
- **Quantum Error Correction:**
  - Protects quantum information from decoherence and noise.
  - Uses redundancy and entanglement to correct errors without measuring quantum states directly.
- **Bloch Sphere Representation:**
  - Visualizes a single qubit as a point on a unit sphere.
  - Demonstrates superposition and quantum state manipulation.

Mathematical Foundations in Quantum Computing

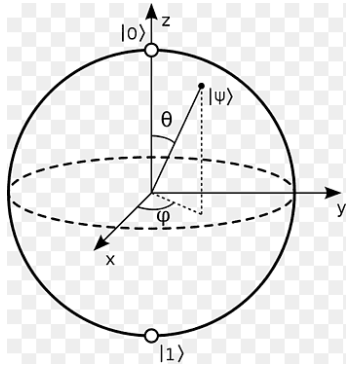

Figure: Bloch Sphere Representation of a Qubit

- Linear algebra: Quantum states are vectors in Hilbert space.
- Complex numbers and unitary operators: Govern quantum gates and evolution.
- Probability and statistics: Measurement outcomes are probabilistic.
- Qubit representation: $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle, \quad |\alpha|^2 + |\beta|^2 = 1.$
- Quantum gates: $U|\psi\rangle$ (unitary transformations).
- Entanglement: $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$.

Artificial Intelligence and Machine Learning

- **What is AI and Machine Learning?**
  - AI refers to the simulation of human intelligence in machines.
  - Machine learning (ML) is a subset of AI that enables computers to learn patterns from data.
- **Applications:**
  - AI-powered chat systems for customer support and health consultations.
  - Predictive analytics for forecasting trends in various industries.
  - Optimization algorithms used in robotics and automation.

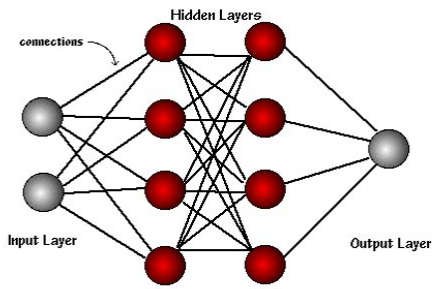

Figure: Neural Network Diagram in AI and Machine Learning

Artificial Intelligence and Mathematical Reasoning

- **Key Questions:**
  - How can AI assist in mathematical reasoning?
    - Automating proofs and theorem verification.
    - Symbolic reasoning and problem-solving with AI.
    - AI-based assistants for research (e.g., Lean, Coq, Wolfram Alpha).
  - Can AI write mathematics instead of manually writing formulas?
    - AI for LaTeX and automated equation generation.
    - AI-driven symbolic computation (e.g., Mathematica, SymPy).
  - What are the mathematical foundations of AI?
    - Linear algebra: Vectors, matrices, and transformations.
    - Probability theory: Bayesian models and uncertainty quantification.
    - Optimization: Gradient descent and loss minimization.
  - How does AI work internally?
    - **Architecture:** Structure of neural networks and deep learning models.
    - **Backpropagation:** The core algorithm for training neural networks.
    - **Hardware:** Role of GPUs and TPUs in accelerating AI computations.

Mathematical Biology: Genome Assembly

- **What is Genome Assembly?**
  - Reconstructing a genome from fragmented DNA sequences.
  - Critical for personalized medicine, evolutionary studies, and biodiversity research.
- **Why is it Mathematically Interesting?**
  - Involves NP-hard optimization problems.
  - Heavy reliance on graph theory: Eulerian circuits and Hamiltonian paths.
- **Mathematical Foundations:**
  - De Bruijn graphs:
    - Nodes represent k-mers (substrings of length k).
    - Edges represent overlaps of $k-1$ bases.
  - Coverage probability equation:

$$P = 1 - \left(1 - \frac{\ell}{G}\right)^n,$$

where ℓ = read length, G = genome size, n = number of reads.

Outcome

- **Key Outcomes of the Seminar:**
  - Understanding of Mathematical Foundations across different disciplines (MANE, Quantum Computing, AI/ML).
  - Exploration of Real-World Applications in engineering, computation, and modeling.
  - Introduction to Key Equations and Methods such as Navier-Stokes, Quantum Gates, and Neural Networks.
  - Interdisciplinary Problem-Solving through discussions and mathematical analysis.
  - Preparation for Advanced Topics & Research Directions.

Thank You!

Questions? Let's Discuss!

Website: https://mtalha086.github.io/graduate$_r$*eading*$_s$*eminar*/
Join Our Discord: https://discord.gg/StX35NbJ